Mission Function Control of Tethered Subsatellite Deployment/Retrieval: In-Plane and Out-of-Plane Motion

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I. Introduction

MANY tethered subsatellite systems are proposed for various future applications including the Shuttle-based "skyhook" concept.[1-3] Dynamics of deployment/retrieval of such systems is time varying and nonlinear, and its control is rather complicated. Many studies are reported in the area of control and dynamics of a tethered subsatellite.[1] A control algorithm called the mission-function control is applied successfully to control deployment and retrieval of a tethered subsatellite.[2] The dynamics treated in Ref. 2 are restricted to a case where the tether is assumed to swing only in the orbital plane.

This Note extends the numerical analysis in Ref. 2. Both in-plane and out-of-plane motions are taken into account for a dynamical model. Results of the numerical analysis show an excellent controlled behavior of the mission-function control algorithm, even for the present dynamical model.

II. Equations of Motion

A tethered system model treated in this Note is illustrated in Fig. 1. The center of mass of the main body is denoted by C and the center of attraction by P. The orthogonal axes X, Y, and Z are defined along a vector $\boldsymbol{PC}$, a vector along the orbital velocity, and the orbital angular velocity vector, respectively, with origin at C. The parameters m, l, Θ, and ϕ denote mass of the subsatellite, length of the tether, position angle of the subsatellite in the orbital plane (in-plane rotation) and that in the out-of-plane rotation, respectively.

The dynamical model is simplified by making the following assumptions:

1) The external force affecting the motion is only the gravitational force due to P. The orbit is circular with a radius R_0 and a constant angular velocity Ω.

2) The mass of the subsatellite is sufficiently small with respect to mass of the main body that C moves in a circular orbit around P.

3) The tether has no mass and thus its flexibility is negligible.

4) The control force acts only through tether tension T, and no control force or energy dissipation exists for motion perpendicular to the tether line.

The equations describing motion of the present system are obtained in the nondimensional form as follows:

$$\Lambda'' - \Lambda\phi'^2 - \Lambda\Theta'^2\cos^2\phi - 2\Lambda\Theta'\cos^2\phi - 3\Lambda\cos^2\Theta\cdot\cos^2\phi + \Lambda\sin^2\phi = -\hat{T} \tag{1a}$$

$$\Lambda\Theta''\cos\phi + 2\Lambda'\Theta'\cos\phi - 2\Lambda\Theta'\phi'\sin\phi + 2\Lambda'\cos\phi - 2\Lambda\phi'\sin\phi + 3\Lambda\sin\Theta\cdot\cos\Theta\cdot\cos\phi = 0 \tag{1b}$$

$$\Lambda\phi'' + 2\Lambda'\phi' + \Lambda\Theta'^2\sin\phi\cdot\cos\phi + 2\Lambda\Theta'\sin\phi\cdot\cos\phi + 3\Lambda\cos^2\Theta\cdot\sin\phi\cdot\cos\phi + \Lambda\sin\phi\cdot\cos\phi = 0 \tag{1c}$$

where $(\ )' = d(\ )/d\tau$, $\tau = \Omega t$, t is time; $\Lambda = l/|l_0 - l_m|$ with l_0 and l_m denoting the initial and the desirable tether length for the deployment or retrieval, respectively; nondimensionalized tension is denoted by $\hat{T} = T/(m\Omega^2|l_0 - l_m|)$.

It may be noted that Eqs. (1) are nonlinear and time varying since these contain the time-varying parameters l and $\dot{l}$.

III. Mission Function Control

The present problem of the deployment and retrieval of a subsatellite is described by the following mission. To change the dynamical state of the system from an initial state to a desired one (call it the mission state)

$$\Lambda = \Lambda_m,\ \Lambda' = \Theta' = \Theta = \phi' = \phi = 0 \tag{2}$$

where Λ_m is the nondimensionalized length at the mission state. The mission function is selected as

$$M = (1/2)\,[a_1\Lambda'^2 + a_2(\Lambda - \Lambda_m)^2 + b_1(\Theta'^2\cos^2\phi + \phi'^2 + 3\sin^2\Theta\cdot\cos^2\phi + 4\sin^2\phi)] \tag{3}$$

where a_1, a_2, and b_1 are positive weighting coefficients. As is apparent from Eq. (3), the mission function M is positive definite and zero at the mission state, Eq. (2). The dynamical state approaches to the mission state when the differentiation of the mission function with respect to the nondimensional time is set to be negative definite.[2] Using Eqs. (1), the nondimensional time derivative of the mission function is obtained as follows:

$$\frac{dM}{d\tau} = -a_1\Lambda'\tilde{T} \tag{4}$$

where

$$\hat{T} = \Lambda(\phi'^2 + \Theta'^2\cos^2\phi + 2\Theta'\cos^2\phi + 3\cos^2\Theta\cdot\cos^2\phi - \sin^2\phi) + (a_2/a_1)(\Lambda - \Lambda_m) - 2(b_1/a_1)(\Theta'^2\cos^2\phi + \Theta'\cos^2\phi + \phi'^2)/\Lambda + \tilde{T} \tag{5}$$

and $\tilde{T}$ is the freely assignable part of $\hat{T}$.

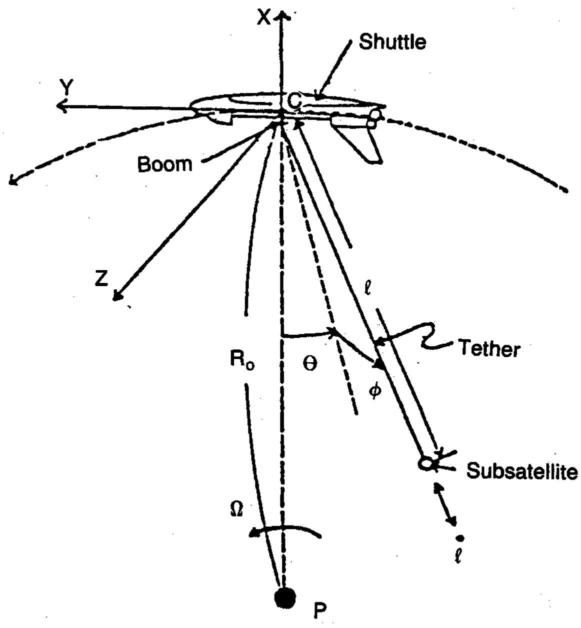

Fig. 1 Schematic representation of a tethered satellite system.

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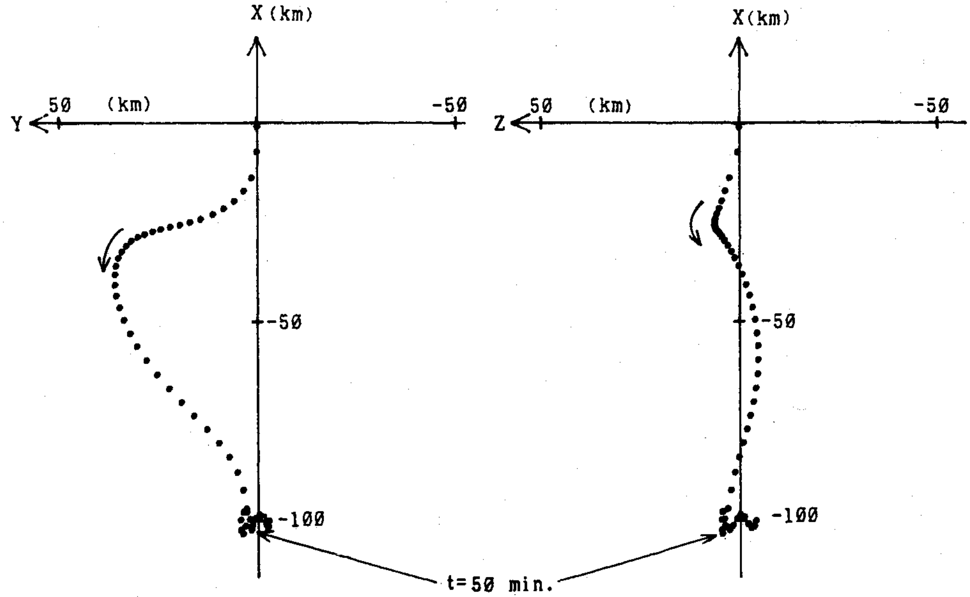
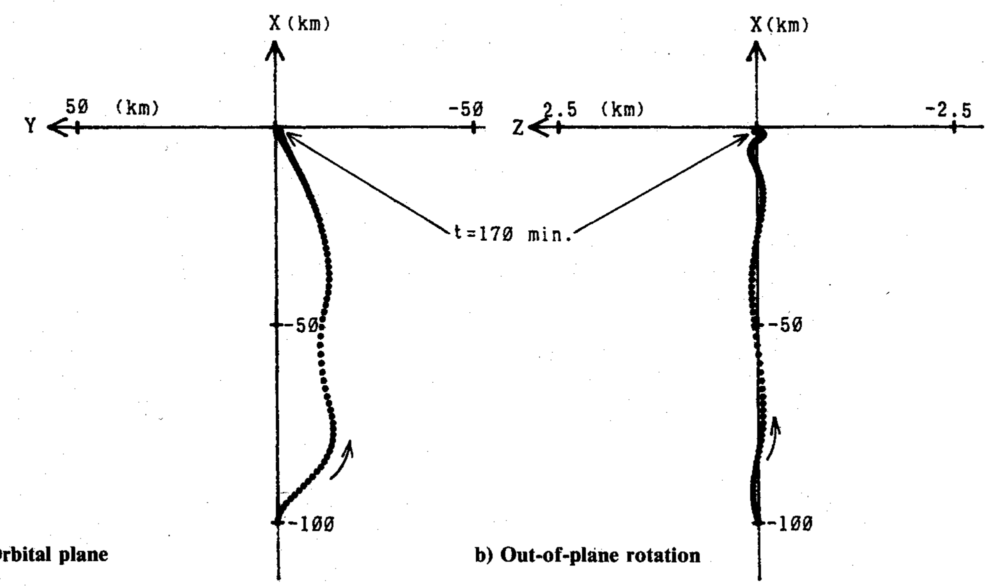


a) Orbital plane b) Out-of-plane rotation

Fig. 2 Time history of deployment of a subsatellite in three dimensional motion: $a_1 = 0.6$; $a_2 = 50$; $b_1 = 1.0$; $k = 40$.



a) Orbital plane b) Out-of-plane rotation

Fig. 3 Time history of retrieval of subsatellite in three dimensional motion: $a_1 = 0.6$; $a_2 = 10$; $b_1 = 3.0$; $k = 20$.

Selection of $\tilde{T}$ in this case as

$$\tilde{T} = 2(k/a_1)\Lambda\Lambda' M \quad (k>0) \tag{6}$$

gives a representation of $dM/d\tau$ as follows:

$$\frac{dM}{d\tau} = -2k\Lambda\Lambda'^2 M \tag{7}$$

The time derivative of the mission function is apparently negative definite, as shown in Eq.(7). The tension, $\hat{T}$, through use of Eqs.(5) and (6), gives the control algorithm for the tethered system.

IV. Numerical Results

Control of deployment and retrieval of a subsatellite swinging in-plane and out-of-plane motions is studied, and results of the numerical simulation are shown in Figs. 2 and 3. An initial velocity of the out-of-plane rotation is added to disturb the out-of-plane motion of the tether. Otherwise, the out-of-plane rotation will stay at rest. The main body is assumed to follow a circular orbit with radius 6600 km and with orbital angular velocity 4.048 deg/min.

The deployment problem is set with the initial conditions:

$$l_0 = 1 \text{ km}, \; l_0' = 15.6 \text{ km/min}, \; \dot{\phi}_0 = 286 \text{ deg/min},$$

$$\Theta_0 = \dot{\Theta}_0 = \phi_0 = 0 \tag{8}$$

and with the mission state:

$$l_m = 100 \text{ km}, \quad l_m' = \Theta_m = \dot{\Theta}_m = \phi_m = \dot{\phi}_m = 0 \tag{9}$$

where subscripts 0 and m denote those values at the initial state and the mission state, respectively.

Figures 2 show the time history during deployment of the subsatellite. The effect of the Coriolis force is apparent in Fig. 2a, particularly until about 21 min after the start of the process, and the present control algorithm is seen to be very effective. Both the in-plane rotation Θ and the out-of-plane rotation ϕ converge gradually to 0 deg as shown in Figs. 2. The object of this case is accomplished after 50 min in the process.

The initial conditions and mission state in the retrieval case are set, respectively, by the following equations:

$$l_0 = 100\ \text{km}, \quad \dot{\Theta}_0 = 5.73\times 10^{-3}\ \text{deg/min},$$

$$\dot{l}_0 = \Theta_0 = \phi_0 = \dot{\phi}_0 = 0 \qquad (10)$$

$$l_m = 1\ \text{km}, \quad \dot{l}_m = \Theta_m = \dot{\Theta}_m = \phi_m = \dot{\phi}_m = 0 \qquad (11)$$

The time history during retrieval of the subsatellite is shown in Fig. 3. It takes about 170 min to accomplish the control objective. The tether length l in-plane rotation Θ and out-of-plane rotation ϕ are 0.365 km, 25.5 deg, and −0.755 deg, respectively, at that time.

V. Conclusions

The control problem of deployment and retrieval of the tethered subsatellite has been studied by using the mission function control.

The results of numerical simulation show that controllability of the mission function control is affirmed on the deployment and retrieval of the subsatellite connected to a main body through a tether swinging both in in-plane and out-of-plane motions. It is therefore concluded that the present control algorithm works quite well during this control problem when the three-dimensional motion of the tethered subsatellite is taken into account for the analysis.

References

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Fast Orbit Propagator for Graphical Display

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Introduction

It is often desirable to provide a graphical display of spacecraft orbits, either as ground tracks on flat maps or as paths on perspective views of the globe. The resolution of wall displays or personal computer screens does not demand the accuracy provided by a sophisticated orbit propagator, and so a fast, simple, analytic two-body propagator is suitable. Naive applications of analytic propagators that ignore the distinction between mean and osculating Keplerian elements can lead to large intrack errors. We present a simple analytic orbit propagator incorporating a mean-to-osculating transformation and study its errors in a numerical example.

Formulation

An extended-time, zeroth order theory[1] computes orbits with sufficient accuracy for our purpose. That is, in propagating an orbit from time t_0 to time t, we can tolerate errors of order J_2, but not of order $J_2(t - t_0)$. This means that we must compute the secular perturbations and the semimajor axis (which affects the mean motion) to first order in J_2, but we can ignore other orbit perturbations. The transformations between Cartesian elements and osculating Keplerian elements are standard, so the orbit propagation problem is to compute the osculating Keplerian elements at time t in terms of the osculating Keplerian elements at time t_0. This proceeds in three steps: conversion of osculating to mean Keplerian elements at time t_0, propagation of the mean elements from t_0 to t, and conversion of mean to osculating Keplerian elements at time t.

Consider the propagation of the mean elements first. Denoting the Brouwer mean elements by overbars and retaining only the secular terms in J_2, we find that the semimajor axis $\bar{a}$, eccentricity $\bar{e}$, and inclination $\bar{i}$ are constant, and that[2]

$$\bar{\omega}(t) = \bar{\omega}(t_0) + (3/2)J_2R^2p^{-2}\bar{n}[2 - (5/2)\sin^2\bar{i}](t - t_0) \qquad (1)$$

$$\bar{\Omega}(t) = \bar{\Omega}(t_0) - (3/2)J_2R^2p^{-2}\bar{n}\cos\bar{i}(t - t_0) \qquad (2)$$

$$\bar{M}(t) = \bar{M}(t_0) + \bar{n}(t - t_0) \qquad (3)$$

where

$$p \equiv \bar{a}(1 - \bar{e}^2) \qquad (4)$$

$$\bar{n} = \sqrt{\mu/\bar{a}^3}[1 + (3/2)J_2R^2p^{-2}(1 - \bar{e}^2)^{1/2}(1 - (3/2)\sin^2\bar{i})] \qquad (5)$$

In these equations, R is the equatorial radius of the Earth, used only for scaling J_2, and μ is the Earth's gravitational constant. It is convenient to define a new "mean" semimajor axis $\hat{a}$ by

$$\hat{a} \equiv \bar{a}[1 - J_2R^2p^{-2}(1 - \bar{e}^2)^{1/2}(1 - (3/2)\sin^2\bar{i})] \qquad (6)$$

since this and Eq. (5) give

$$\bar{n} = \sqrt{\mu/\hat{a}^3} \qquad (7)$$

to first order in J_2. Note that $\hat{a}$, like $\bar{a}$, is independent of time. We can also replace p in Eq. (1) and (2) by

$$\hat{p} \equiv \hat{a}(1 - \bar{e}^2) \qquad (8)$$

since p (the semilatus rectum) only appears in terms already first order in J_2 in these equations. Thus, the propagation of the mean elements is given by Eq. (3),

$$\bar{\omega}(t) = \bar{\omega}(t_0) + (3/2)J_2R^2\hat{p}^{-2}\bar{n}(2 - 5/2)\sin^2\bar{i}](t - t_0) \qquad (9)$$

$$\bar{\Omega}(t) = \bar{\Omega}(t_0) - (3/2)J_2R^2\hat{p}^{-2}\bar{n}\cos\bar{i}(t - t_0) \qquad (10)$$

Now consider the conversion of mean to osculating elements at time t. We can ignore the difference between mean and osculating values of all Keplerian elements except the semimajor axis because the errors introduced by this approximation are of order J_2. However, we must compute the mean-to-osculating transformation of the semimajor axis to order J_2 because an order J_2 error in the semimajor axis would give, through Eq. (7), an order J_2 error in the mean motion $\bar{n}$, and thus, an error of order $J_2(t - t_0)$ in the orbit. The difference between the mean and osculating semimajor axis is a short-period term (Ref. 2, p. 288),

$$a = \bar{a} + (J_2R^2/a)\{(1 - (3/2)\sin^2 i)[(a/r)^3 - (1 - e^2)^{-3/2}] + (3/2)(a/r)^3\sin^2 i\cos 2(f + \omega)\} \qquad (11)$$

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